

Fab 5: Noncanonical Kinetic Gravity, Self Tuning, and Cosmic Acceleration

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We investigate circumstances under which one can generalize Horndeski's most general scalar-tensor theory of gravity. Specifically we demonstrate that a nonlinear combination of purely kinetic gravity terms can give rise to an accelerating universe without the addition of extra propagating degrees of freedom on cosmological backgrounds, and exhibit self tuning to bring a large cosmological constant under control. This nonlinear approach leads to new properties that may be instructive for exploring the behaviors of gravity.

I. INTRODUCTION

Theories of gravitation on cosmic scales have become an area of intense interest, both as a possible explanation for the observed cosmic acceleration and as an exploration of consistent extensions of general relativity. Generically such extensions lead to additional degrees of freedom, e.g. scalar modes in scalar-tensor theories, with possible pitfalls of higher than second order derivative field equations that may lack a well posed initial value formulation, or of ghosts and other instabilities.

Horndeski in 1974 wrote the most general scalar-tensor theory giving second order field equations in four dimensional spacetimes [1]. In an alternate view, Galileon theories [2–4], shift symmetric scalar fields possessing nonlinear combinations of field derivatives, have recently been studied with interest as sound models capable of cosmic acceleration. Also recently, as an approach to solve the cosmological constant problem, a linear combination of four terms called the Fab Four has been identified [5–7] as the unique terms allowing self tuning vacua that can cancel a large bare Λ term.

Here we draw on aspects of all three of these approaches to demonstrate that nonlinear combinations of terms involving shift symmetric scalar fields can possess interesting advantages and properties. This extension of the “most general” scalar-tensor theory retains second order field equations and avoids pathologies on symmetric spacetimes such as the usual Friedmann-Lemaître-Robertson-Walker (FLRW) and de Sitter cosmologies. While the nonlinear approach can be applied quite generally, we give a proof of principle using a simple example of purely kinetic couplings with noncanonical forms, extending the “purely kinetic gravity” of [8].

Our example employs a nonlinear combination of the standard kinetic term and derivative coupling to the Einstein tensor. Besides possessing second order field equations it does not add any further propagating degrees of freedom, and it can achieve lasting cosmic acceleration unlike the linear, canonical, purely kinetic gravity theory of [8], avoid at least some instabilities unlike the derivatively coupled Galileon theory investigated by [9], and self tune away a cosmological constant like the Fab Four.

Deeper implications exist beyond our simple proof of principle. We emphasize that the example given is intended purely as a proof of principle to inspire further investigation into the theoretical properties of general nonlinear combinations, and not as a fit to observations.

In Sec. II we explain our nonlinear generalization procedure and the conditions under which no additional propagating degrees of freedom are generated. The equations of motion are solved in Sec. III on a FLRW background, giving the cosmic and field evolution complete with attractors, revealing two distinct ways of approaching a de Sitter asymptotic state. Section IV demonstrates the self tuning properties of the theory, erasing an initial cosmological constant. The perturbed equations in Sec. V yield the no-ghost and stability conditions and the evolution of the effective Newton's constant G_{eff} . We discuss various implications of the results in Sec. VI.

II. PROMOTION TO NONLINEAR FUNCTION

The Einstein-Hilbert action of general relativity is extremely simple, condensing all the gravitational influence into the Ricci scalar curvature R . To allow cosmic acceleration, however, one must add either a cosmological constant Λ or additional degrees of freedom such as a scalar field ϕ , e.g. with a potential and canonical kinetic term $X \equiv (-1/2)g^{\mu\nu}\phi_\mu\phi_\nu$ where $\phi_\mu = \nabla_\mu\phi$. The cosmological constant, or the field potential, raises issues of fine tuning and

naturalness: why don't high energy radiative corrections affect the form and magnitude to something characteristic of the early universe? We therefore do not employ either (except in Sec. IV where we erase them).

A canonical kinetic term cannot by itself give rise to acceleration but noncanonical (but still minimally coupled) kinetic terms can, called k-essence [10–12], or in the absence of any potential, purely kinetic k-essence (e.g. [13]). One can think of this as promoting the Lagrangian term linear in the canonical kinetic contribution to a function. This generically gives an extra degree of freedom, in that the sound speed is no longer fixed to the speed of light.

Alternately, one could promote the Ricci scalar term to a function, hence $f(R)$ theories [14]. This again adds a degree of freedom and one can view these as coupled scalar-tensor theories. Similarly one can have theories involving the Gauss-Bonnet combination $G_{GB} = R^{abcd}R_{abcd} - 4R^{ab}R_{ab} + R^2$ of the Riemann tensor, Ricci tensor, and Ricci scalar, either linearly in G_{GB} or promoted to a function [15, 16]. In generalized Galileon theories one can promote the coefficients of the standard Galileon terms to functions of the canonical kinetic term, for example [17, 18].

Taking these examples as motivation, suppose we take the Horndeski theory action, composed of the linear combination of several terms, and instead promote them to nonlinear functions or combinations. In generality we cannot do this without resulting in higher than second order equations of motion or adding unconstrained degrees of freedom. However, in specific circumstances we can. For example a theory involving a function of the Ricci scalar and Gauss-Bonnet term $f(R, G_{GB})$ can be sound [19]. In Fab Four terminology, this mixes George and Ringo (though we do not allow the field potentials). This is permissible because of particular symmetries within these terms.

In purely kinetic gravity theories, similar symmetries impose a unique Lagrangian involving only the Einstein tensor coupled to the field derivatives [20] (which Fab Four term John basically replicates). Up to mass dimension 6, the action is just the linear combination of the canonical kinetic term and the Einstein coupled kinetic term [8], effectively giving a disformal field theory. This could achieve transient cosmic acceleration but not an asymptotic de Sitter state, and was later shown to have ghosts [9]. Allowing for an arbitrary constant coefficient of the canonical kinetic term, one could achieve a de Sitter asymptote but the theory has an early time Laplace instability [9].

Merging these two approaches of nonlinear function promotion and purely kinetic terms of great simplicity, we examine as a specific example nonlinear functions of the canonical and the Einstein coupled kinetic terms. The combination of nonlinearity and noncanonical nature delivers new characteristics to the theory. Since this “hip-hop” kinetic evolution extends the Fab Four self tuning possibilities, among other properties, we call this new Lagrangian term Fab 5 Freddy. As the line “Fab 5 Freddy told me everybody’s fly” from Blondie’s *Rapture* [21] predicts, this term also enables cosmic acceleration and an asymptotic de Sitter behavior, indeed in multiple ways.

The action we study in detail is

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R + c_1 X + f \left(c_2 X + \frac{c_G}{M^2} G^{\mu\nu} \phi_\mu \phi_\nu \right) \right] + S_m, \quad (1)$$

where $G^{\mu\nu}$ is the Einstein tensor associated to the metric $g_{\mu\nu}$, S_m represents the action for the matter fields, and M is a mass scale to keep c_G dimensionless, where we normalize to $M = H_0$. When the function f is linear then this is the derivatively coupled Galileon (using only \mathcal{L}_2 in [9]), generalizing the purely kinetic gravity model by allowing a free constant coefficient for the canonical term.

To study the effects of the nonlinear promotion we consider two cases: 1) $c_1 = 0$, so the canonical and Einstein coupled kinetic terms are directly coupled nonlinearly, and 2) $c_2 = 0$, so only the derivative coupling appears nonlinearly. This allows us to compare these two different theories with the same linear limit.

We can rewrite the action in terms of a Lagrange multiplier field χ , as

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R + c_1 X + f(\chi) + \left(c_2 X + \frac{c_G}{M^2} G^{\mu\nu} \phi_\mu \phi_\nu - \chi \right) \frac{df}{d\chi} \right] + S_m. \quad (2)$$

Varying the action (2) in terms of χ we find

$$\left(c_2 X + \frac{c_G}{M^2} G^{\mu\nu} \phi_\mu \phi_\nu - \chi \right) \frac{d^2 f}{d\chi^2} = 0. \quad (3)$$

This has the solution

$$\chi = c_2 X + \frac{c_G}{M^2} G^{\mu\nu} \phi_\mu \phi_\nu, \quad (4)$$

except at particular points for which $f_{\chi\chi} = 0$ (and note that in the linear case χ is moot). Subscripts χ denote derivatives with respect to χ . By re-inserting the solution Eq. (4) back into Eq. (2), we verify that we obtain the original action Eq. (1).

Introducing a Lagrange multiplier field χ helps understanding of the independent degrees of freedom. In particular, such a Lagrange multiplier can be coupled with other elements (such as the Ricci scalar, as in the $f(R)$ theories, or the Einstein tensor, as in our case). Both the Ricci scalar and the Einstein tensor are functions of a second derivative of the metric. Therefore by integrating by parts, a time-derivative for the Lagrange multiplier may appear. In this case, such a Lagrange multiplier can in general acquire a kinetic term, and it may start propagating. This situation, as already said, is common to those theories which can be written in terms of a Lagrange multiplier coupled to a second-order operator, e.g. as in $f(R)$ or $R + f(G_{\text{GB}})$. The theory $f(R, G_{\text{GB}})$ introduces two Lagrange multipliers. This theory is quite interesting as it has been proven that only one of these two new scalar degrees of freedom will propagate on Friedmann-Lemaître-Robertson-Walker backgrounds [19]. On the other hand, both these degrees of freedom do propagate on anisotropic backgrounds. Therefore whether or not these Lagrange multipliers propagate or not depends on the chosen theory.

We will see that the theory at hand, Eq. (1), will not introduce on cosmological backgrounds any new degree of freedom. However, we will find that the high- k limit (where k is the wavemode) dispersion relation of perturbations will be modified, leading to a scale-dependent speed of propagation, i.e. $c_s^2 \propto k^2$. This is indeed similar to what happens in the $f(R, G_{\text{GB}})$ case. On a formal level it will be interesting to study eventually our theory on anisotropic backgrounds to see whether or not the Lagrange multiplier will start propagating and we will discuss this issue in a future project.

III. EQUATIONS OF MOTION AND EVOLUTION

We give the general covariant background equations of motions in Appendix A. Here we specialize to a homogeneous and isotropic spacetime where the metric is FLRW. The theory then has the property that the equations of motion for the action remain second order. We include a barotropic fluid (i.e. matter and radiation) with energy density ρ and pressure P and assume spatial flatness for simplicity. The background equations of motion for the action of Eq. (2) are then

$$3M_{\text{pl}}^2 H^2 = \rho + \frac{1}{2}c_1\dot{\phi}^2 + \frac{1}{2}c_2f_\chi\dot{\phi}^2 + f_\chi\chi - f + 9\frac{c_G}{M^2}f_\chi H^2\dot{\phi}^2, \quad (5)$$

$$2\left(M_{\text{pl}}^2 - \frac{c_G}{M^2}f_\chi\dot{\phi}^2\right)\dot{H} = -P - 3M_{\text{pl}}^2 H^2 - \frac{1}{2}c_1\dot{\phi}^2 - f - \left(\frac{1}{2}c_2\dot{\phi}^2 - 3\frac{c_G}{M^2}H^2\dot{\phi}^2 - \chi - 4\frac{c_G}{M^2}H\dot{\phi}\ddot{\phi}\right)f_\chi + 2\frac{c_G}{M^2}H\dot{\phi}^2\dot{f}_\chi, \quad (6)$$

$$\left(c_1 + c_2f_\chi + 6\frac{c_G}{M^2}H^2f_\chi\right)\ddot{\phi} = -3c_1H\dot{\phi} - 12\frac{c_G}{M^2}f_\chi\dot{\phi}H\dot{H} - (\dot{f}_\chi + 3Hf_\chi)\left(c_2 + 6\frac{c_G}{M^2}H^2\right)\dot{\phi}, \quad (7)$$

$$\chi = \left(\frac{1}{2}c_2 + 3\frac{c_G}{M^2}H^2\right)\dot{\phi}^2, \quad (8)$$

$$\dot{\rho} = -3H(\rho + P), \quad (9)$$

where $H = \dot{a}/a$ is the Hubble expansion rate of the scale factor a .

Linear perturbations about the background are important for calculating the growth of structure, which we consider in Sec. V, but also for analyzing the degrees of freedom. Details of the equations are given in Appendix B but here we note a key point. The coupled system of perturbed equations for the two metric potentials, the barotropic fluid density and velocity, the ϕ scalar field, and the Lagrange multiplier scalar field χ does not possess any time derivatives $\delta\chi$. This indicates that χ is merely an auxiliary field with no dynamics but rather an algebraic constraint, and is uniquely determined by the other fields. This arises because the Einstein tensor within $f(\chi)$ only depends on first derivatives and not second derivatives in the Robertson-Walker background, i.e. only H^2 appears.

To obtain the solutions to the evolution of the expansion H and field ϕ , we put the background equations in the form of an autonomous system of coupled equations, using the dimensionless parameters $\bar{H} \equiv H/H_0$ and $x \equiv \phi'/M_{\text{pl}}$, where primes denote derivatives with respect to $N = \ln a$. Then

$$x' = \frac{\lambda\gamma - \omega\alpha}{\alpha\sigma - \lambda\beta} \quad (10)$$

$$\bar{H}' = -\frac{\gamma}{\alpha} - \frac{\beta}{\alpha}x' \quad (11)$$

where

$$\alpha = 2\bar{H} - 6f_\chi c_G \bar{H}^3 x^2 - 2c_G f_{\chi\chi} \bar{H}^4 x^2 (c_2 \bar{H} x^2 + 12c_G \bar{H}^3 x^2) \quad (12)$$

$$\beta = -2c_G f_{\chi\chi} \bar{H}^4 x^2 (c_2 \bar{H}^2 x + 6c_G \bar{H}^4 x) - 4f_\chi c_G \bar{H}^4 x \quad (13)$$

$$\gamma = 3\bar{H}^2 + f_\chi \left(\frac{c_2}{2} \bar{H}^2 x^2 - \chi - 3c_G \bar{H}^4 x^2 \right) + \frac{\Omega_{r0}}{a^4} + f + \frac{c_1}{2} \bar{H}^2 x^2 \quad (14)$$

$$\sigma = c_2 f_\chi \bar{H}^2 + 6f_\chi c_G \bar{H}^4 + f_{\chi\chi} \bar{H}^2 x (c_2 + 6c_G \bar{H}^2) (c_2 \bar{H}^2 x + 6c_G \bar{H}^4 x) + c_1 \bar{H}^2 \quad (15)$$

$$\lambda = f_\chi c_2 \bar{H} x + 18f_\chi c_G \bar{H}^3 x + f_{\chi\chi} \bar{H}^2 x (c_2 + 6c_G \bar{H}^2) (c_2 \bar{H} x^2 + 12c_G \bar{H}^3 x^2) + c_1 \bar{H} x \quad (16)$$

$$\omega = 3f_\chi \bar{H}^2 x (c_2 + 6c_G \bar{H}^2) + 3c_1 \bar{H}^2 x \quad (17)$$

with Ω_{r0} the dimensionless radiation energy density today and

$$\chi = \frac{c_2}{2} \bar{H}^2 x^2 + 3c_G \bar{H}^4 x^2. \quad (18)$$

To ensure the accuracy of our numerical solution we use as a check the constraint equation (5), written in the dimensionless parameters as

$$\bar{H}^2 = \frac{\Omega_{m0}}{a^3} + \frac{\Omega_{r0}}{a^4} + \frac{1}{3} \left(f_\chi \chi - f + 9f_\chi c_G \bar{H}^4 x^2 + f_\chi \frac{c_2}{2} \bar{H}^2 x^2 + \frac{c_1}{2} \bar{H}^2 x^2 \right), \quad (19)$$

with Ω_{m0} the dimensionless matter density today. The quantity in parentheses can be viewed as an effective dark energy density. An effective dark energy pressure can similarly be defined using Eq. (6), with the effective dark energy equation of state ratio $w_\phi = P_\phi/\rho_\phi$.

A. Early Time Evolution

As in [9], one can identify the early and late time asymptotic solutions. At early times, during radiation or matter domination, when $\bar{H}^2 \gg 1$, one generally has $\chi \approx 3c_G \bar{H}^4 x^2$ and $\Omega_\phi \ll 1$ (if one fine tunes the c_1 or c_2 terms to dominate instead then the energy density would decay as $\rho_\phi \sim a^{-6}$ and hence be uninteresting). In this case the solution becomes

$$x \sim a^{3[1+3w_b+4e(1+w_b)]/[2(1+2e)]} \quad (20)$$

$$\chi \sim a^{-3(1-w_b)/(1+2e)}, \quad (21)$$

where w_b is the barotropic equation of state (0 for matter domination, 1/3 for radiation domination), and $e \equiv \chi f_{\chi\chi}/f_\chi$. To go further we must adopt a specific form for the function f . Taking $f(\chi) = A\chi^n$, we have $e = n - 1$ and

$$\rho_\phi \sim a^{-3n(1-w_b)/(2n-1)} \quad (22)$$

$$w_\phi = \frac{1 - n(1 + w_b)}{2n - 1}. \quad (23)$$

While in the linear model ($n = 1$), the dark energy traces the matter during matter domination, this is not so in the nonlinear model. The case $n = 0$ is a cosmological constant. Note that the dark energy is phantom (and has a ghost, we will later find) for $0 < n < 1/2$. This means that to avoid violation of early radiation/matter domination the field would have to be highly fine tuned, more so than a cosmological constant. For $n \approx 1/2$, the evolution diverges ($e = -1/2$) and matter/radiation domination is violated. Note that this rules out functions that act like $n = 1/2$ power laws at early times, such as a DBI type $f = \sqrt{1 + \chi} - 1$. Therefore we concentrate on $n > 1/2$.

B. Late Time Evolution to de Sitter State

During its evolution, the model leads to cosmic acceleration near the present and an asymptotic de Sitter state. Interestingly, this can arise in two ways. For $\bar{H}' = 0$ and also $x' = 0$ as a fixed point one needs $\gamma = 0$ and either $\omega = 0$ or $\alpha = 0$. Combining the expression for γ with Eq. (19) leads to the condition

$$\bar{H}^2 x^2 [c_1 + f_\chi (c_2 + 6c_G \bar{H}^2)] = 0. \quad (24)$$

This guarantees that $\omega = 0$ also. In the $c_1 = 0$ case, this is the same de Sitter point $\bar{H}_{\text{dS}}^2 = -c_2/(6c_G)$ as in [9] and exists irrespective of the functional form of $f(\chi)$ (as long as $f_\chi \neq 0$ at the de Sitter point). Note that c_2 and c_G must have opposite signs for this de Sitter point to be present. In the $c_2 = 0$ case, there is a new de Sitter point $\bar{H}_{\text{dS}}^2 = -c_1/(6f_\chi c_G)$.

Since for the $c_1 = 0$ de Sitter point we have $\chi \rightarrow 0$, we should choose a function f such that $f(0) = 0$ otherwise we are putting in a cosmological constant. But then no solution for this de Sitter point exists for the nonlinear power law model that simultaneously satisfies $\bar{H} \rightarrow \text{const}$ and $\rho_\phi \rightarrow \text{const} \neq 0$. Recall that $f_\chi \sim \chi^{n-1}$.

However, there is yet another de Sitter solution that we can construct for our nonlinear model; this arises because of the evolution of x such that asymptotically $x' \neq 0$. This solution still has $\bar{H}_{\text{dS}}^2 = -c_2/(6c_G)$ but $x \sim a^{3(n-1)/(3n-1)}$. Thus as $\bar{H} \rightarrow \text{const}$, x decays to 0 for $n < 1$ while x diverges for $n > 1$.

In Fig. 1 we exhibit \bar{H}^2 and ρ_ϕ for the $c_1 = 0$ cases with $n = 1.5$ and $n = 0.8$. As noted, the $n > 1$ case grows quickly relative to the background components and so must start with a small $(\rho_\phi/\rho_m)_i$ to preserve later matter domination. At $a = 10^{-6}$, say, this ratio must be less than $10^{-6.6}$, but this is still not as severe as the cosmological constant fine tuning which requires 10^{-16} . The $n = 0.8$ case can actually dominate over matter at $a = 10^{-6}$, but has rather drastic evolution at $z \approx 1$ as it suddenly turns toward the de Sitter attractor.

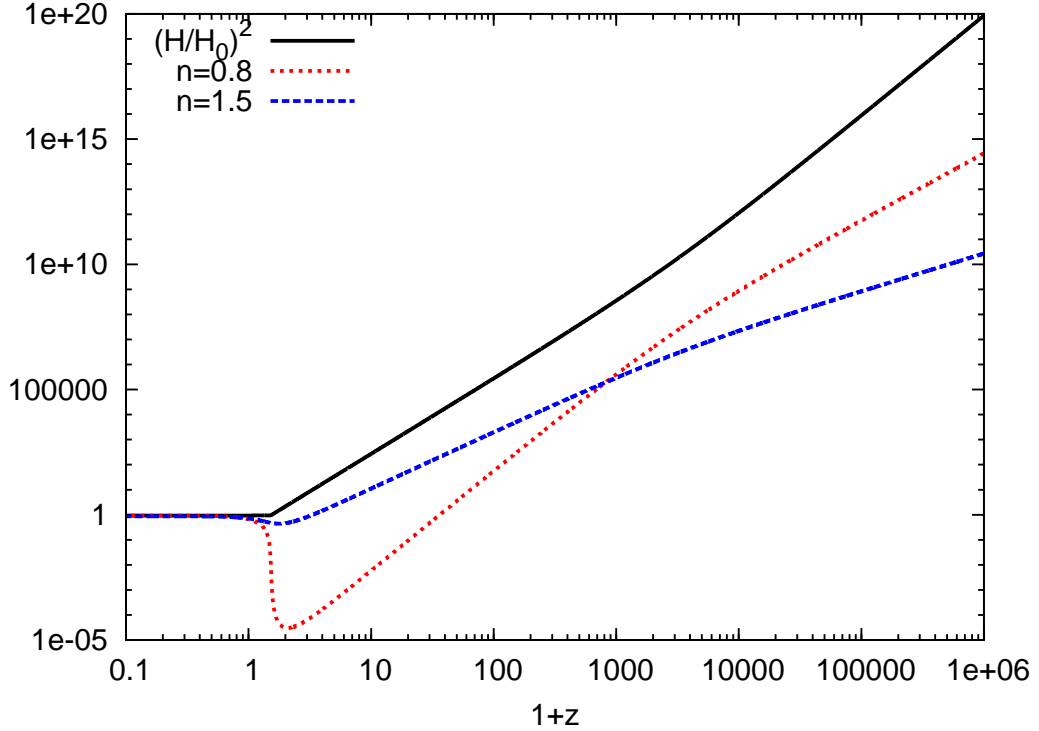


FIG. 1: The evolution of the effective dark energy density ρ_ϕ in the $c_1 = 0$ case is plotted for power law functions $f \sim \chi^n$ with $n = 0.8$ (dotted red curve) and $n = 1.5$ (dashed blue). The expansion history $(H^2/H_0)^2$ (solid black) is also shown (for $n = 0.8$ though the $n = 1.5$ case is nearly identical on this scale). Note that for $n < 1$, typically ρ_ϕ must be set to large values initially so that it does not decay to too small values at late times; conversely, for $n > 1$ ρ_ϕ grows relative to matter and radiation and must be set to low values initially.

For the $c_2 = 0$ case, as mentioned we expect at early times no significant change to the dynamics since again the c_G term will dominate over c_1 . At late times, since ρ_ϕ contains terms with different powers of x there is no extra de Sitter solution (hence $\rho_\phi = \text{const}$) with varying x . Thus the only de Sitter solution is $c_1 + 6f_\chi c_G \bar{H}^2 = 0$. Note that χ freezes at a finite value and so $f \rightarrow \text{const}$. The energy density evolution looks quite similar to the $c_1 = 0$ case and so instead we show the evolution of the dark energy equation of state for the two cases in the first panel of Fig. 2. The spike in $w_\phi(z)$ can be ameliorated by raising the initial field energy density, but as discussed above this would impinge on matter domination.

Although both cases reach de Sitter attractors asymptotically, the manner in which they achieve this differs. For the nonlinearity only applying to the Einstein tensor coupled kinetic term ($c_2 = 0$ case), the solution is the double fixed point $\bar{H}' = 0 = x'$. This holds as well for the full nonlinearity (applied to both kinetic terms, i.e. the $c_1 = 0$ case) when $n > 1$ (but this is not the stable attractor). In addition the full nonlinearity case also has a de Sitter

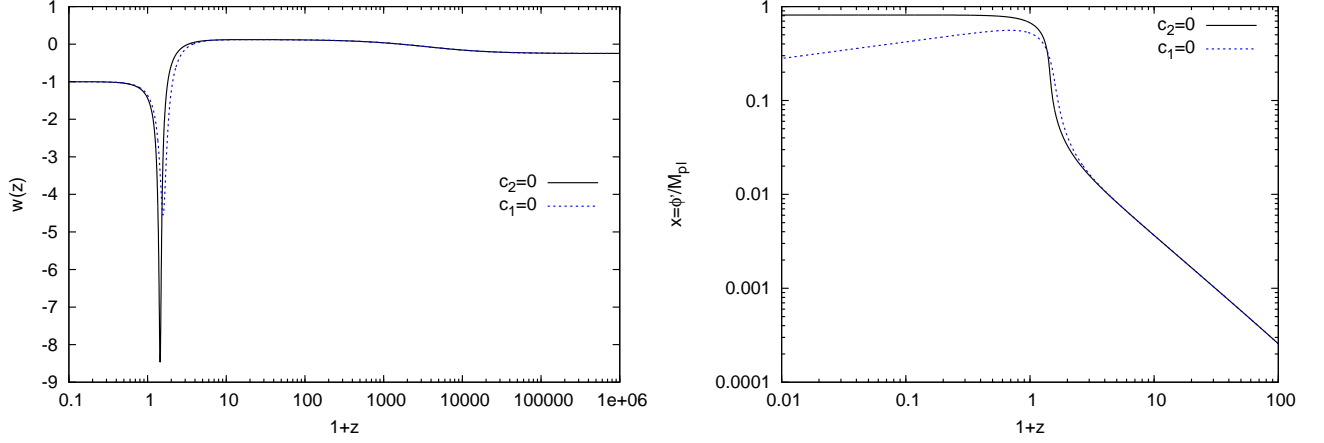


FIG. 2: Comparison of the cases $c_2 = 0$ (solid black) and $c_1 = 0$ (dotted blue), both with $n = 0.9$ and $[\rho_\phi/\rho_m](a = 10^{-6}) = 10^{-3}$, is plotted for the equation of state parameter (left panel) and the field evolution $x = \phi'/M_{\text{pl}}$ (right panel) of the effective dark energy. The $c_2 = 0$ case has a more extreme phantom feature in $w(z)$ near the present, but both cases have the same early time tracking and late time de Sitter attractor (although for $c_1 = 0$ one has $x' \neq 0$).

solution with $\bar{H}' = 0$ but $x' \neq 0$, i.e. $\ddot{\phi} \neq 0$. The field will decelerate, $\ddot{\phi} < 0$ (accelerate, $\ddot{\phi} > 0$) for $n < 1$ ($n > 1$). The difference between the field evolutions for the two cases is shown in the second panel of Fig. 2, for $n = 0.9$.

A summary of the de Sitter attractors is given in Table I, including results from Sec. V on the ghost and Laplace stability conditions of the perturbations.

(c_1, c_2)	n	H_{ds}^2	x_{ds}	χ_{ds}	No-ghost	Laplace
$(0, c_2)$	$1/2 < n < 1$	$-c_2/(6c_G)$	0	0	✓	$\sim \checkmark$
$(0, c_2)$	$n > 1$	$-c_2/(6c_G)$	∞	∞	×	×
$(c_1, 0)$	$1/2 < n < 1$	$-c_1/(6c_G f_\chi)$	const	const	✓	$\sim \checkmark$
$(c_1, 0)$	$n > 1$	$-c_1/(6c_G f_\chi)$	const	const	×	×

TABLE I: Summary of de Sitter attractors is given for the two cases of the model, with a nonlinear function $f \sim \chi^n$. The two cases have different approaches to de Sitter, that merge in the common linear limit $n = 1$.

IV. SELF TUNING

The ability of the theory to reach a de Sitter asymptotic state without a cosmological constant is interesting, as is the overall expansion behavior of such a cosmological model, but more significant is the ability of Fab 5 Freddy to self tune, in the manner of John or Paul in the Fab Four [5]. This allows the scalar field ϕ – even without a potential – to cancel an existing (large) cosmological constant. This even holds if the cosmological constant readjusts as it passes through phase transitions. Here we present a simplified analysis showing these key properties while neglecting matter or radiation components.

The dynamical equations are identical to Eqs. (10)-(18) except for the replacement of $\Omega_{r0} a^{-4}$ by $-3\Omega_\Lambda$ in the γ term, coming from (three times) the background pressure. (Note $\Omega_\Lambda \neq 0.7$, the observed cosmological constant, but is instead the early universe, bare cosmological constant.) Two de Sitter points can be found, both of which are attractors. The first arises from the explicit cosmological constant, with $\bar{H}_1^2 = 8\pi G\rho_\Lambda/(3H_0^2) = \Omega_\Lambda$ and the second is the self tuning solution with $\bar{H}_2^2 = -c_2/(6c_G)$, as we found in the absence of a cosmological constant. Note $\bar{H}_1 \gg \bar{H}_2$.

For the first solution, the scalar field contribution dies away as $\rho_\phi \sim x^{2n} \sim a^{-6n/(2n-1)}$, so the pure cosmological constant is a fixed point of the dynamics. (Of course matter and radiation would also redshift away.) For the second solution the scalar field dynamically adjusts such that $\rho_\phi \rightarrow -\rho_\Lambda$. Note that unlike in the earlier sections $\rho_\phi < 0$. However the same approach to a de Sitter state occurs, with ρ_ϕ dynamically canceling ρ_Λ and retaining a small positive residual energy density, evolving with $x \sim a^{-3(n-1)/(3n-1)}$ on approach to $\bar{H}_2^2 \rightarrow -c_2/(6c_G)$.

These analytic behaviors are verified numerically in Figs. 3 and 4. We include a cosmological constant $\Omega_\Lambda = 10^8$ throughout the numerical calculation, and consider $f(\chi) = -\chi^n$ (adopting $n = 1.5$, $c_G = 1$, $c_2 = -5.6$, $c_1 = 0$). We begin \bar{H} away from both asymptotic solutions $\bar{H} = \bar{H}_{1,2}$, illustrating the behaviors for different initial conditions in

Figs. 3 and 4.

In the left panel of Fig. 3 we observe the approach to the standard cosmological constant attractor with $\bar{H}^2 \rightarrow 8\pi G\rho_\Lambda/(3H_0^2)$, with the right panel showing the vanishing $\rho_\phi \rightarrow 0$.

However, below a certain critical initial condition \bar{H}_i (depending on the other parameters), we observe entirely different dynamical behavior. Now, \bar{H} approaches the second asymptotic point $\bar{H}_2^2 = -c_2/(6c_G)$. This occurs despite the large cosmological constant present in the model. We find that the absolute value of the ϕ field energy density approaches the $\rho_\phi \sim -\rho_\Lambda$ solution, canceling the vacuum energy in the field equations. Hence the model exhibits self tuning, for some range of initial conditions (e.g. $\bar{H}^2(a = 10^{-6}) \lesssim 10^6 \bar{H}_2^2$ for the parameters adopted in the figure).

To see how self tuning occurs, we must examine the equations of motion (here taking $c_1 = 0$)

$$3M_{\text{pl}}^2 H^2 = \frac{1}{2}c_2 f_\chi \dot{\phi}^2 + f_\chi \chi - f + 9\frac{c_G}{M^2} f_\chi H^2 \dot{\phi}^2, \quad (25)$$

$$\left(c_2 + 6\frac{c_G}{M^2} H^2\right) f_\chi \ddot{\phi} = -12\frac{c_G}{M^2} f_\chi \dot{\phi} H \dot{H} - (\dot{f}_\chi + 3H f_\chi) \left(c_2 + 6\frac{c_G}{M^2} H^2\right) \dot{\phi}, \quad (26)$$

$$\chi = \left(\frac{1}{2}c_2 + 3\frac{c_G}{M^2} H^2\right) \dot{\phi}^2. \quad (27)$$

On-shell (that is, at the asymptotic de Sitter state), we have $\bar{H}^2 = -c_2/(6c_G)$ and $\dot{H} = 0$, and hence the scalar field equation is trivially satisfied, carrying no information regarding the evolution of ϕ . However, the scalar field equation contains an explicit \ddot{a} dependence, and the Hamiltonian density \mathcal{H} in the Friedmann equation retains $\dot{\phi}$ dependence on shell, both of which are conditions given in [6] for self tuning to occur. On approach to the de Sitter point, the scalar field continues to evolve while ρ_ϕ and \bar{H} approach constant values.

If we choose initial conditions for \bar{H}_i such that it is initially far from the attractor $\bar{H}_2^2 = -c_2/(6c_G)$, then one can use Eqs. (10) and (11) to calculate how the model approaches the de Sitter state. For the power law models $f(\chi) \sim \chi^n$, we find that the dynamical behaviour of \bar{H} is independent of n , and the evolution toward de Sitter has $\bar{H} - \bar{H}_{\text{dS}} \sim a^{-3}$, $x \sim a^6$. Ultimately the evolution of \bar{H} will depend on the presence of matter and radiation (which we have neglected here), and also the functional form of $f(\chi)$. Whether a specific self tuning model can be constructed that gives rise to a viable cosmological evolution will be the subject of future work (see [7] for the Fab Four case).

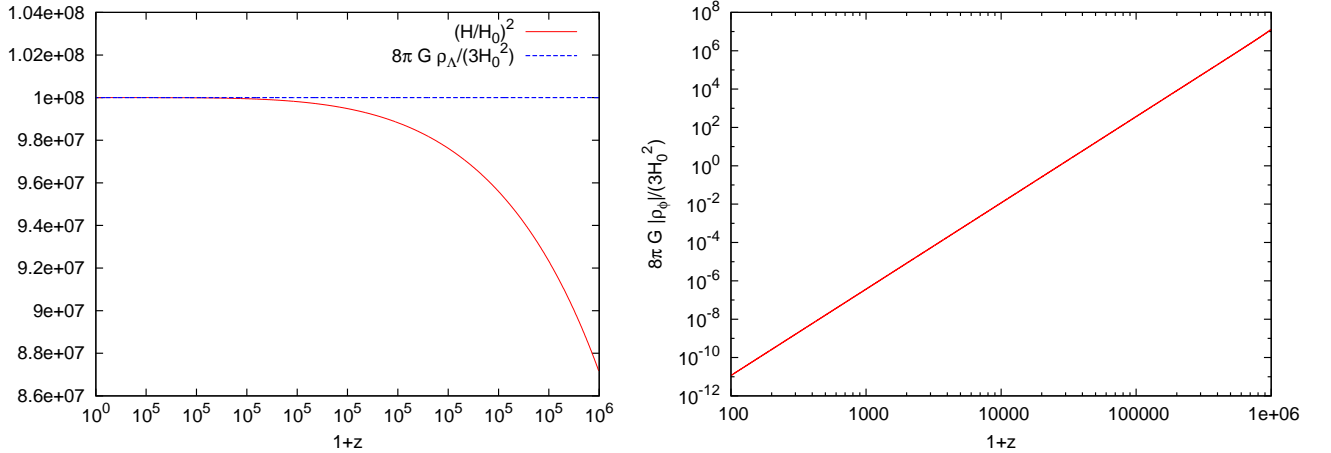


FIG. 3: [Left panel] \bar{H}^2 evolves toward its standard cosmological constant attractor $\bar{H}_1^2 = 8\pi G\rho_\Lambda/(3H_0^2)$ for high density initial conditions, here $\bar{H}_i = 10^4 \sqrt{-c_2/(6c_G)}$. [Right panel] Meanwhile the scalar field energy density decays away.

Going further, we can verify that the self tuning also self adjusts if the vacuum energy undergoes a phase transition at some redshift. We numerically model such an energy density with a tanh function, and choose the pressure to solve the continuity equation $P = -\frac{\rho'}{3} - \rho$. The evolution of the quantities \bar{H}^2 , ρ_ϕ , and Ω_Λ are shown in Figs. 5 and 6, demonstrating that the two de Sitter solutions still hold and the self tuning mechanism remains effective. The explicit cosmological constant can be made effectively invisible in our model.

V. LINEAR PERTURBATIONS

Linear perturbations of the equations of motion are important for calculating the growth of structure and assessing the ghost-free and stability conditions of the theory. For subhorizon perturbations one adopts the quasistatic approx-

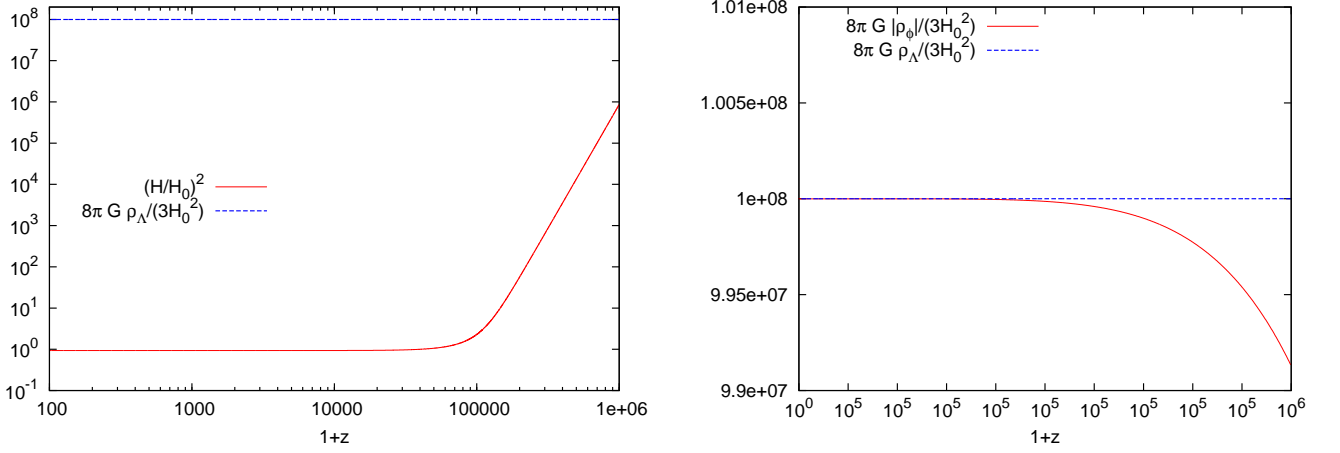


FIG. 4: As Fig. 3, but with initial conditions $\bar{H}_i = 10^3 \sqrt{-c_2/(6c_G)}$. [Left panel] Now we observe the dynamics leads to the second attractor $\bar{H}^2 \rightarrow -c_2/(6c_G)$, despite the presence of a large cosmological constant. [Right panel] The scalar field energy density ρ_ϕ self tunes to cancel the ρ_Λ contribution in the field equations.

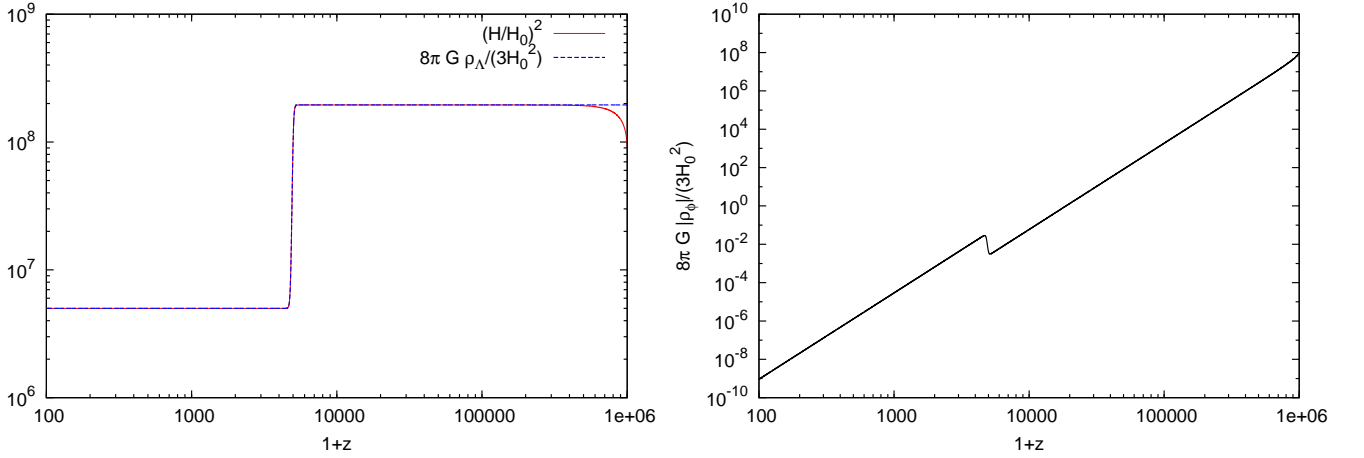


FIG. 5: As Fig. 3 for $\bar{H}_i = 10^4 \sqrt{-c_2/(6c_G)}$, but with the large vacuum energy undergoing a phase transition. The standard attractor $\bar{H}^2 \rightarrow 8\pi G\rho_\Lambda/(3H_0^2)$ applies, and the ϕ field energy density asymptotically decays, subject to a mild jump at the phase transition.

imation. We begin by using Eq. (B2) to write the equations of motion for the perturbation in the Newtonian gauge ($\beta = 0$ in Appendix B) as follows:

$$2M_{\text{pl}}^2 \nabla^2 \Phi = \rho_m \delta_m + 9 \frac{c_G}{M^2} f_{\chi\chi} H^2 \dot{\phi}^2 \delta\chi + \frac{c_G}{M^2} f_\chi \left(2\dot{\phi}^2 \nabla^2 \Phi - 4H\dot{\phi} \nabla^2 \delta\phi \right) + f_{\chi\chi} \chi \delta\chi + \frac{c_2}{2} f_{\chi\chi} \dot{\phi}^2 \delta\chi \quad (28)$$

$$\delta\chi = 2 \frac{c_G}{M^2} \dot{\phi}^2 \nabla^2 \Phi \quad (29)$$

$$c_1 \nabla^2 \delta\phi + c_2 f_\chi \nabla^2 \delta\phi - c_2 \dot{\phi} (f_{\chi\chi} \delta\chi) + 2 \frac{c_G}{M^2} f_\chi (2\dot{H} + 3H^2) \nabla^2 \delta\phi - 6 \frac{c_G}{M^2} H^2 \dot{\phi} (f_{\chi\chi} \delta\chi) - c_2 f_{\chi\chi} (\ddot{\phi} + 3H\dot{\phi}) \delta\chi \quad (30)$$

$$- 6 \frac{c_G}{M^2} f_{\chi\chi} H^2 \ddot{\phi} \delta\chi - 6 \frac{c_G}{M^2} f_{\chi\chi} H (2\dot{H} + 3H^2) \dot{\phi} \delta\chi - 4 \frac{c_G}{M^2} f_\chi \ddot{\phi} \nabla^2 \Phi - 4 \frac{c_G}{M^2} \dot{f}_\chi \dot{\phi} \nabla^2 \Phi + 4 \frac{c_G}{M^2} f_\chi H \dot{\phi} (\nabla^2 \psi - \nabla^2 \Phi) = 0$$

$$\begin{aligned} \partial_i \partial_j \Phi - \partial_i \partial_j \psi + g_{ij} (\nabla^2 \psi - \nabla^2 \Phi) &= \frac{c_G}{M^2} f_\chi \left\{ 2(\ddot{\phi} + H\dot{\phi}) (g_{ij} \nabla^2 \delta\phi - \partial_i \partial_j \delta\phi) + \dot{\phi}^2 [g_{ij} (\nabla^2 \Phi + \nabla^2 \psi) - (\partial_i \partial_j \Phi + \partial_i \partial_j \psi)] \right\} \\ &+ g_{ij} \left(\frac{c_2}{2} f_{\chi\chi} \dot{\phi}^2 \delta\chi - \frac{c_G}{M^2} \left[2H\dot{\phi}^2 (f_{\chi\chi} \delta\chi) + f_{\chi\chi} (4H\dot{\phi} \ddot{\phi} + 2\dot{H}\dot{\phi}^2 + 3H^2\dot{\phi}^2) \delta\chi \right] - f_{\chi\chi} \chi \delta\chi \right) \\ &+ \frac{c_G}{M^2} \dot{\phi}^2 f_{\chi\chi} (\partial_i \partial_j \delta\chi - g_{ij} \nabla^2 \delta\chi) + 2 \frac{c_G}{M^2} \dot{f}_\chi \dot{\phi} (g_{ij} \nabla^2 \delta\phi - \partial_i \partial_j \delta\phi) . \end{aligned} \quad (31)$$

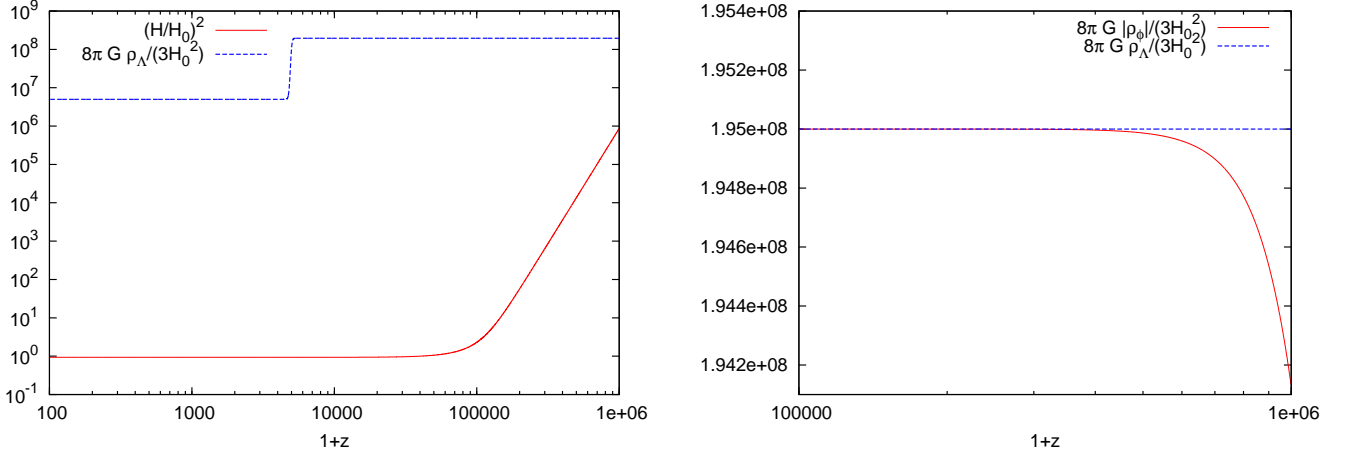


FIG. 6: As Fig. 5 with a large vacuum energy undergoing a phase transition, but with $\bar{H}_i = 10^3 \sqrt{-c_2/(6c_G)}$. Self tuning remains effective despite the transition and the second attractor $\bar{H}^2 \rightarrow -c_2/(6c_G)$ is approached. The ϕ field dynamically adjusts energy density to cancel the vacuum energy, and the transition does not modify the Hubble parameter due to it already being on the attractor.

There are some important differences between this case and the linear case where $f_{\chi\chi} = 0$, i.e. no nonlinear mixing. Here, in the (i, j) Einstein and ϕ field equations, terms appear of the form $k^4 \Phi f_{\chi\chi}$ and $k^2 \dot{\Phi} f_{\chi\chi}$, arising from $\delta\chi$. These will lead to scale dependence in the gravitational coupling strength G_{eff} derived below. Recall that the standard Galileon case does not have scale dependent coupling on cosmic scales well above the Vainshtein scale (see G_{eff} from [9]).

A. Evolution of Gravity

To investigate the modified Poisson equations defining the coupling of matter to the metric potentials, we can use the $(i, j = i)$ perturbed Einstein equation to remove $(f_{\chi\chi} \delta\chi)$, and then substitute for the $(i, j \neq i)$, ϕ , and χ equations. In the quasistatic limit appropriate for linear growth on subhorizon scales the $(0, 0)$ perturbed Einstein equation becomes

$$\nabla^2 \Phi = 4\pi a^2 G_{\text{eff}}^{(\Phi)} \rho_m \delta_m. \quad (32)$$

The equivalent modified Poisson equations for the other metric potential combinations are

$$\nabla^2 \psi = 4\pi G_{\text{eff}}^{(\psi)} \rho_m \delta_m \quad (33)$$

$$\nabla^2 (\Phi + \psi) = 8\pi G_{\text{eff}}^{(\Phi+\psi)} \rho_m \delta_m. \quad (34)$$

The gravitational couplings are

$$\frac{G_{\text{eff}}^{(\Phi)}}{G_N} = \frac{\kappa_3 \kappa_8 + 2\kappa_2 \kappa_9}{\kappa_1 (\kappa_3 \kappa_8 + 2\kappa_2 \kappa_9) + \kappa_2 (\kappa_5 \kappa_8 + 2\kappa_2 \kappa_7 - \kappa_4 \kappa_8 \kappa_6)}, \quad (35)$$

$$\frac{G_{\text{eff}}^{(\psi)}}{G_N} = - \left[\frac{\kappa_9}{\kappa_8} \left(\frac{\kappa_1 \bar{G}_{\text{eff}}^{(\Phi)} - 1}{\kappa_2} \right) + \frac{\kappa_7}{\kappa_8} \bar{G}_{\text{eff}}^{(\Phi)} \right] \quad (36)$$

$$\frac{G_{\text{eff}}^{(\Phi+\psi)}}{G_N} = \left(\frac{\kappa_8 - \kappa_7}{2\kappa_8} \right) \bar{G}_{\text{eff}}^{(\Phi)} - \frac{\kappa_9}{2\kappa_8} \left(\frac{\kappa_1 \bar{G}_{\text{eff}}^{(\Phi)} - 1}{\kappa_2} \right), \quad (37)$$

where $\bar{G}_{\text{eff}}^{(\Phi)} = G_{\text{eff}}^{(\Phi)} / G_N$ and

$$\kappa_1 = 1 - 12c_G^2 \bar{f}_{\chi\chi} \bar{H}^6 x^4 - c_2 c_G \bar{f}_{\chi\chi} \bar{H}^4 x^4 - c_G \bar{f}_{\chi} \bar{H}^2 x^2 \quad (38)$$

$$\kappa_2 = -2c_G \bar{f}_{\chi} \bar{H}^2 x \quad (39)$$

$$\kappa_3 = c_1 + c_2 \bar{f}_{\chi} + 2c_G \bar{f}_{\chi} (2\bar{H} \bar{H}' + 3\bar{H}^2) \quad (40)$$

$$\kappa_4 = -c_2 \bar{H} x - 6c_G \bar{H}^3 x \quad (41)$$

$$\kappa_5 = -4c_G \bar{f}_{\chi} \bar{H} (\bar{H} x' + \bar{H}' x + \bar{H} x) - 12c_G^2 \bar{f}_{\chi\chi} \bar{H}^5 x^2 (3\bar{H} x' + 7\bar{H}' x + 3\bar{H} x) - 6c_2 c_G \bar{f}_{\chi\chi} \bar{H}^3 x^2 (\bar{H} x' + \bar{H}' x + \bar{H} x) \quad (42)$$

$$\kappa_6 = -c_G \bar{f}_{\chi\chi} \bar{H} x [x k^2 + \bar{H} (4\bar{H} x' + 6\bar{H}' x + 7\bar{H} x)] \quad (43)$$

$$\kappa_7 = 1 + c_G \bar{f}_{\chi} \bar{H}^2 x^2 - 2c_G^2 \bar{f}_{\chi\chi} \bar{H}^4 x^4 k^2 / a^2 \quad (44)$$

$$\kappa_8 = -1 + c_G \bar{f}_{\chi} \bar{H}^2 x^2 \quad (45)$$

$$\kappa_9 = 2c_G \bar{f}_{\chi} \bar{H} (\bar{H} x' + \bar{H}' x + \bar{H} x) + 2c_G \bar{f}_{\chi} \bar{H}^2 x \quad (46)$$

and all quantities are in dimensionless form, i.e. $\bar{H} = H/H_0$, $\bar{f} = f/(M_{\text{pl}}^2 H_0^2)$, and primes denote derivatives with respect to $N = \ln a$.

In the de Sitter limit for the case $c_1 = 0$, one finds $G_{\text{eff}}/G_N = 1/\kappa_1$. However, although both χ and x approach 0, they do so such that $f_{\chi}\chi \rightarrow 0$, $f_{\chi}x^2 \rightarrow \text{const}$, and $f_{\chi\chi}x^4 \rightarrow \infty$. Thus $|\kappa_7| \gg |\kappa_1| \rightarrow +\infty$ and $G_{\text{eff}} \rightarrow 0$. That is, gravity appears to turn off at late times. This arises in this limit from the nonlinear structure of the theory, i.e. the presence of $f_{\chi\chi}$ and its power law behavior.

The numerical solutions for the evolution $G_{\text{eff}}(z)$ are shown in Fig. 7. At high redshift the theory acts as general relativity, then deviations begin when $(k/aH)^2 \Omega_{\phi}^2 \sim 1$. At this point, the κ_7 contribution to G_{eff} will dominate due to the k^2 term. Since κ_7 appears in the denominator of $G_{\text{eff}}^{(\Phi)}$, this scale dependent effective Newton's constant will typically vanish at high redshift, during matter domination (the exact redshift will be scale dependent and will also be determined by the initial conditions for the scalar field energy density). $G_{\text{eff}}^{(\psi)}$ on the other hand will not vanish at early times owing to its different κ_7 dependence.

The gravitational coupling $G_{\text{eff}}^{(\psi)}$ entering matter growth behaves as GR until near the present, since large κ_7 actually cancels out from it. At low redshift it spikes and then vanishes. The gravitational coupling $G_{\text{eff}}^{(\Phi+\psi)}$ entering light deflection is given by the mean $[G_{\text{eff}}^{(\Phi)} + G_{\text{eff}}^{(\psi)}]/2$ and so shows deviations at both high and low redshift. We emphasize that the current model is not proposed as observationally viable but rather to introduce interesting theoretical properties of nonlinear, noncanonical kinetic gravity.

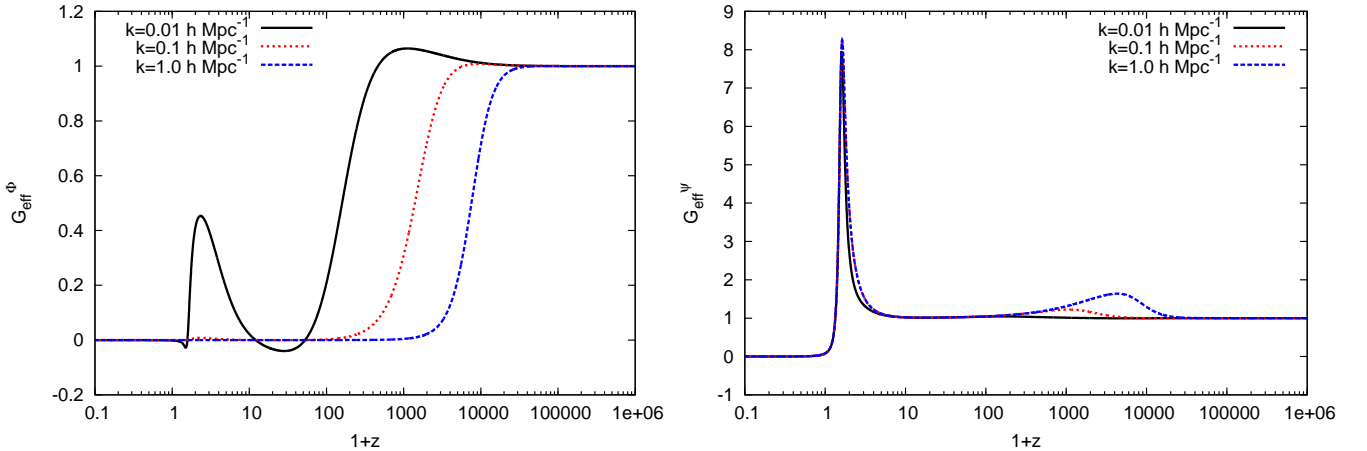


FIG. 7: The gravitational couplings G_{eff} in the nonlinear theory (left panel $G_{\text{eff}}^{(\Phi)}$, right panel $G_{\text{eff}}^{(\psi)}$, both for $f = \chi^{0.9}$) become scale dependent, as shown by the evolution for three different wave modes: $k = 0.01 h/\text{Mpc}$ (solid black), $k = 0.1 h/\text{Mpc}$ (dotted red) and $k = 1.0 h/\text{Mpc}$ (dashed blue). Gravity vanishes at late times.

B. Ghost and Stability Conditions

In order to find the ghost conditions, we need the action for the independent degrees of freedom. It is convenient for this task to evaluate Eq. (B2) in the flat gauge (i.e. $\Phi = 0$). Then we can see that the fields ψ, β and $\delta\chi$ can be integrated out leaving only two scalars to propagate, i.e. $\delta\phi$ (the new-gravity mode), and v (the matter mode). But there is a crucial subtlety: the quadratic term $\delta\chi^2$ will generate a term proportional to $k^4\delta\phi^2/a^4$. This means that this theory will modify the high k behaviour of the modes, and this will lead to possible cosmological signatures. This situation is similar to what happens for FLRW backgrounds in the $f(R, G_{\text{GB}})$ theories [19]. This k^4 -dependent term vanishes when $f_{\chi\chi} = 0$, that is when the action is linear in the combination $c_2 X + (c_G/M^2)G^{\mu\nu}\phi_\mu\phi_\nu$. This behaviour is a typical signature of the presence of a massive mode ($\delta\chi$), whose kinetic term vanishes, but not its mass.

After removing the auxiliary field $\delta\chi$, and this is possible only when $f_{\chi\chi} \neq 0$, we can write down the action as

$$S = \int d^4x a^3 \left[A_{ab} \dot{V}_a \dot{V}_b + \frac{B\epsilon_{ab}}{a^2} (\partial_i \dot{V}_a)(\partial_i V_b) - \frac{D_{ab}}{a^4} (\partial^2 V_a)(\partial^2 V_b) - \frac{E_{ab}}{a^2} (\partial_i V_a)(\partial_i V_b) + C\epsilon_{ab} \dot{V}_a V_b + M_{ab} V_a V_b \right], \quad (47)$$

where we have defined $V_1 = \delta\phi$, $V_2 = v$. The matrices A, D, E, M , as well as the two coefficients B and C , are functions of the background. Here we have also defined ϵ_{ab} as the two dimensional antisymmetric matrix with $\epsilon_{12} = 1$. Furthermore, the only non-zero matrix element of the matrix D corresponds to D_{11} .

The no-ghost requirements are

$$\det[A] = [f_{\chi\chi} \{c_G (c_2 + 6c_G \bar{H}^2) (12c_G \bar{H}^2 - c_2) \bar{H}^4 x^4 f_\chi + \bar{H}^2 x^2 (c_2^2 - 6c_1 c_G^2 \bar{H}^4 x^2 + 36c_G^2 \bar{H}^4 + 12c_2 c_G \bar{H}^2)\} + c_G (18c_G \bar{H}^2 - c_2) \bar{H}^2 x^2 f_\chi^2 + (c_2 + 6c_G \bar{H}^2 - c_1 c_G \bar{H}^2 x^2) f_\chi + c_1] \frac{(1+w)\bar{\rho}_w (1 - c_G \bar{H}^2 x^2 f_\chi)}{4\Delta_2} > 0 \quad (48)$$

$$A_{22} = [(c_G^2 (36c_G \bar{H}^2 + 5c_2) f_\chi \bar{H}^6 x^6 - c_G \bar{H}^4 x^4 (c_1 c_G \bar{H}^2 x^2 + 18c_G \bar{H}^2 + 2c_2)) f_{\chi\chi} + 9c_G^2 \bar{H}^4 x^4 f_\chi^2 - 6c_G \bar{H}^2 x^2 f_\chi + 1] \times \frac{(1+w)\bar{\rho}_w}{2\Delta_2} > 0 \quad (49)$$

where in addition to the scalar field we have assumed the presence of a barotropic fluid with equation of state w and energy density $\bar{\rho}_w = \rho_w/(H_0^2 M_{\text{pl}}^2)$, and

$$\Delta_2 = [\{\bar{H}^6 x^6 c_G^2 (36c_G \bar{H}^2 + 5c_2) f_\chi - c_G^2 \bar{\rho}_w \bar{H}^4 x^4 - c_G \bar{H}^4 x^4 (c_G c_1 \bar{H}^2 x^2 + 18c_G \bar{H}^2 + 2c_2)\} f_{\chi\chi} + 9c_G^2 \bar{H}^4 x^4 f_\chi^2 - 6c_G \bar{H}^2 x^2 f_\chi + 1] w - c_G^2 \bar{\rho}_w \bar{H}^4 x^4 f_{\chi\chi}. \quad (50)$$

During the radiation era, $\Delta_2 \approx w = 1/3$ and the square brackets in A_{22} resolve to 1, so indeed $A_{22} > 0$. In the matter era where $w = 0$, then A_{22} has the same sign as $-f_{\chi\chi}$ and so we require $n < 1$ in the power law model $f \sim \chi^n$. For $\det A$, since $\bar{H}^2 \gg 1$ then the c_G terms will dominate in the early universe over the other scalar field terms in the absence of fine tuning them to be small. This results in the condition $(2n-1)/(1-n) > 0$, satisfied for $1/2 < n < 1$. Thus such theories are free of ghosts.

Checking the speed of propagation of the field, with Laplace stability given by nonnegative sound speed squared, $c_s^2 \geq 0$, is somewhat more involved. In the high- k limit, we find that the dispersion relations are given by

$$\omega_\phi^2 = \frac{B^2 + A_{22} D_{11}}{\det[A]} \frac{k^4}{a^4} = \frac{16c_G^4 f_\chi^2 \bar{H}^8 x^6 f_{\chi\chi}}{(1 - c_G f_\chi \bar{H}^2 x^2) \Delta} \frac{k^4}{a^4}, \quad (51)$$

$$\omega_{\text{pf}}^2 = \frac{D_{11} E_{22}}{B^2 + A_{22} D_{11}} \frac{k^2}{a^2} = w \frac{k^2}{a^2}, \quad (52)$$

where Δ is defined as

$$\Delta \equiv f_{\chi\chi} [c_G (c_2 + 6c_G \bar{H}^2) (c_2 - 12c_G \bar{H}^2) \bar{H}^4 x^4 f_\chi + 6c_1 c_G^2 \bar{H}^6 x^4 - (c_2 + 6c_G \bar{H}^2)^2 \bar{H}^2 x^2] - c_G (18c_G \bar{H}^2 - c_2) \bar{H}^2 x^2 f_\chi^2 + [c_1 c_G \bar{H}^2 x^2 - (c_2 + 6c_G \bar{H}^2)] f_\chi - c_1. \quad (53)$$

The speeds of propagation are then found as the group velocity $c = a\partial\omega/\partial k$, or

$$c_\phi^2 = \frac{64c_G^4 f_\chi^2 \bar{H}^8 x^6 f_{\chi\chi}}{(1 - c_G f_\chi \bar{H}^2 x^2) \Delta} \frac{k^2}{a^2} \geq 0, \quad c_{\text{pf}}^2 = w \geq 0. \quad (54)$$

One of the two solutions is trivial as it corresponds to the speed of the perfect fluid, but the other one sets a stability condition, and states that the speed of propagation will be scale dependent.

Because of the k^4 terms in Eq. (47), the dispersion relation and hence sound speed will be wavenumber dependent. From Eq. (54), we see that in the high k limit the leading order contribution to c_ϕ^2 goes like

$$c_\phi^2 \sim \left(\frac{k}{aH} \right)^2 \Omega_\phi^2 \quad (55)$$

where $\Omega_\phi = 8\pi G\rho_\phi/(3H^2)$. During matter domination, we typically find $k/(aH) \gg 1$ for sub-horizon modes relevant to linear perturbation theory, and $\Omega_\phi \sim \mathcal{O}(10^{-2}\Omega_m)$ (this is largely dependent upon the initial conditions imposed, however this is a conservative upper bound on how large Ω_ϕ can be during matter domination). Hence we expect the k^4 term in Eq. (47) to be the dominant contribution to c_ϕ^2 between $z \sim (1, 1000)$.

If this held for radiation domination, then $c_s^2 \sim (k/aH)^2 \Omega_\phi^2 (1-n)/(2n-1)$ and so the theory would be Laplace stable for $1/2 < n < 1$. However, in the linear case $n = 1$, the terms proportional to $f_{\chi\chi}$ vanish identically and the leading order k^2 contribution vanishes, leaving a scale independent sound speed. As found in [9], the linear theory composed of a standard kinetic term and a kinetic term coupled to the Einstein tensor (i.e. the purely kinetic gravity theory of [8] with the canonical kinetic term generalized to have an arbitrary constant coefficient) is Laplace unstable in the radiation era. Thus, the nonlinearity of the current model can avoid that instability.

However, at early enough times during radiation domination Ω_ϕ drops so low that the $(k/aH)^2 \Omega_\phi^2$ term becomes subdominant for the modes k relevant to linear perturbation theory. To calculate the leading order behaviour of c_ϕ in the very early Universe, it is more instructive to consider the issue from a different angle.

To preserve the CMB acoustic peak structure, and also to obtain an expansion history consistent with observations, we want initial conditions during radiation domination such that the effect of the scalar field ϕ on the background expansion and the metric perturbations is negligible. In this case we can assume that $\Omega_\phi \ll 1$ and the metric potentials are sourced by density perturbations only, hence we can treat the scalar field perturbations $\delta\phi$ as evolving on an otherwise standard cosmological background. Under this assumption, one can analytically calculate the no-ghost and Laplace conditions. For the linear model $f(\chi) \sim \chi$, it was found in [9] that the scalar field perturbations possessed a sound speed c_ϕ^2 that was negative during radiation domination; $c_\phi^2 = -1/3$. This would lead to exponential growth of the scalar field perturbations, destroying the standard cosmological picture. One can perform a similar analysis for the more general $f(\chi)$ case. We find the following no-ghost and Laplace stability conditions

$$3c_G \bar{H}^2 (f_\chi + 2\chi f_{\chi\chi}) > 0 \quad (56)$$

and

$$\frac{2\dot{H} + 3H^2}{3H^2} \frac{f_\chi}{f_\chi + 2\chi f_{\chi\chi}} > 0 \quad (57)$$

where we assume that at early times the c_G contribution to the ϕ energy density is much larger than the standard canonical c_2 term (which is valid barring an extreme fine tuning of c_G). For the power law models, during radiation domination these conditions correspond to

$$c_G f_\chi < 0 \quad (58)$$

$$2n - 1 < 0, \quad (59)$$

which would violate the positivity of ρ_ϕ . This could potentially cause problems in the transition to the matter dominated era, where $\rho_\phi > 0$ is required to ensure that the no-ghost condition is satisfied. We conclude that the power law models cannot simultaneously satisfy the no-ghost and Laplace stability requirements at all times while also having $\rho_\phi > 0$ during radiation domination. This does not preclude the possibility that a non-power law model might be constructed that can.

The presence of a scale-dependent speed of propagation is a feature of this model and it has physical implications, especially at late times when $\Omega_\phi \rightarrow 1$. The reason for the presence of such a term may be due to the large symmetries of the FLRW manifolds, similarly to what happens in the context of $f(R, G_{GB})$ theory (see e.g. [19]). In that case it was shown that the kinetic term of one of the scalar perturbation modes was vanishing on general FLRW manifolds, so that it could be integrated out from the Lagrangian, giving rise in this way to a scale dependent speed of propagation $c_s^2 \propto k^2/a^2$. Furthermore, also in that theory, at late times as the k^2 -regime starts dominating, gravity for high k 's tends to become weaker and weaker, i.e. $G_{\text{eff}}/G_N \rightarrow 0$. It would be interesting to study, along the same lines of the $f(R, G_{GB})$ theories, whether anisotropic models (such as Bianchi-I type manifolds) possess more propagating degrees of freedom than FLRW. This will be investigated in a future project. Nonetheless, if indeed this scenario does happen, then this theory would behave similarly also to massive gravity, as it was shown that for that theory the kinetic terms of three perturbation modes vanish on FLRW due to the high degree of symmetries of the background [22].

VI. CONCLUSIONS

Gravitation is a fundamental force that we have just begun to explore cosmologically. One of the great advances made in gravity research in the past few years is the realization that symmetry principles both strongly restrict the theory and open up new avenues and effects. Galileon gravity and massive gravity both use shift symmetric fields and their couplings to functions of the metric to enable new properties, including cosmic acceleration without a cosmological constant or field potential. An action allowed by the symmetries and well behaved in initial value formulation, specifically one leading to second order equations of motion, is of particular interest. If moreover the field exhibits self tuning, allowing it to overcome a high energy cosmological constant, the theory is well worth examining.

We show that by promoting a purely kinetic gravity term to a nonlinear function, possibly mixed with a noncanonical kinetic term for the field, fascinating properties can ensue. In addition to second order equations of motion and self tuning, the theory does not incur extra propagating degrees of freedom on a cosmological background. Similar effects of symmetric backgrounds are seen in massive gravity. For example, in massive gravity it was found [22] that in isotropic spacetimes the shift symmetric (Stückelberg) fields have vanishing mixing between the graviton and the scalar mode, and furthermore their kinetic terms vanish.

The new term discussed here, “Fab 5 Freddy,” can self accelerate and is merely the harbinger of a whole class of such nonlinear promotions or combinations of terms.

The background evolution of the expansion and field lead to early time tracker behavior and late time de Sitter attractors. Solving the linear perturbation equations we see that simple power law functions can be free of ghosts. The gravitational coupling and dispersion relation of perturbations become scale dependent, possibly leading to an early time instability and a late time vanishing of gravity. The specific models studied may not be observationally viable but the characteristics arising from the nonlinear, noncanonical action open new aspects of gravity. Most intriguing is the self tuning property that can cancel a bare cosmological constant dynamically, even through phase transitions. The evolution of the field basically makes Λ invisible.

That the most general scalar-tensor theory giving second order field equations in 4D could be further generalized, at least on cosmological backgrounds, is highly interesting. The specific term considered here, a nonlinear promotion of the field kinetics coupled to the Einstein tensor (the unique, low mass dimension shift symmetric combination giving second order field equations), is merely a proof of principle, while theoretically instructive. Ways to extend this class of theory more generally, to different nonlinear functions and combinations, are straightforward and may preserve the most interesting and desirable characteristics while leading to more viable predictions experimentally.

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Appendix A: Covariant Equations of Motion

The scalar field equation is given by

$$c_1 \square \phi + c_2 \nabla_\alpha [f_\chi \nabla^\alpha \phi] - 2c_G G^{\alpha\beta} \nabla_\alpha [f_\chi \nabla_\beta \phi] = 0 . \quad (\text{A1})$$

The χ field is given by

$$\chi = -\frac{c_2}{2} \nabla_\alpha \phi \nabla^\alpha \phi + c_G G^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi . \quad (\text{A2})$$

The Einstein equations are given by

$$G_{\mu\nu} = 8\pi G \left[T_{\mu\nu}^{(\text{mat})} + T_{\mu\nu}^{(\text{rad})} + T_{\mu\nu}^{(\phi)} \right] , \quad (\text{A3})$$

where

$$\begin{aligned}
T_{\mu\nu}^{(\phi)} = & -c_G f_\chi \left[g_{\mu\nu} \square \phi \square \phi - 2 \square \phi \nabla_\mu \nabla_\nu \phi + 2 \nabla_\mu \nabla_\lambda \phi \nabla_\nu \nabla^\lambda \phi - g_{\mu\nu} \nabla_\lambda \nabla_\alpha \phi \nabla^\lambda \nabla^\alpha \phi \right] \\
& + c_G f_\chi \left[R_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi + R \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} R \nabla_\alpha \phi \nabla^\alpha \phi \right] \\
& - 2 c_G f_\chi \left[R_{\lambda\nu} \nabla^\lambda \phi \nabla_\mu \phi + R_{\lambda\mu} \nabla^\lambda \phi \nabla_\nu \phi - g_{\mu\nu} R_{\rho\lambda} \nabla^\rho \phi \nabla^\lambda \phi + R^\sigma{}_{\mu\beta\nu} \nabla^\beta \phi \nabla_\sigma \phi \right] \\
& + 2 c_G \left[(\nabla_\alpha \nabla_{(\mu} f_\chi) \nabla_{\nu)} \phi \nabla^\alpha \phi - \frac{1}{2} (\square f_\chi) \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla_\alpha \nabla_\beta f_\chi) \nabla^\alpha \phi \nabla^\beta \phi - \frac{1}{2} (\nabla_\mu \nabla_\nu f_\chi) \nabla_\alpha \phi \nabla^\alpha \phi + \frac{1}{2} g_{\mu\nu} (\square f_\chi) \nabla_\alpha \phi \nabla^\alpha \phi \right] \\
& + 2 c_G \left[\nabla_{(\mu} f_\chi \nabla_{\nu)} \phi \square \phi - \nabla_{(\mu} f_\chi \nabla_{\nu)} \nabla_\alpha \phi \nabla^\alpha \phi - \nabla_\alpha f_\chi \nabla^\alpha \nabla_{(\mu} \phi \nabla_{\nu)} \phi - g_{\mu\nu} \nabla^\alpha f_\chi \nabla_\alpha \phi \square \phi + g_{\mu\nu} \nabla^\alpha f_\chi \nabla^\beta \phi \nabla_\alpha \nabla_\beta \phi \right. \\
& \left. + \nabla_\alpha f_\chi \nabla^\alpha \phi \nabla_\mu \nabla_\nu \phi \right] + g_{\mu\nu} f - g_{\mu\nu} f_\chi \chi + c_1 \left[\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi \right] + c_2 f_\chi \left[\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi \right], \tag{A4}
\end{aligned}$$

and parentheses in a subscript denote symmetrization of the indices.

Appendix B: Perturbation Equations in Detail

1. The scalar perturbations

Let us write down the perturbed metric in the following form

$$ds^2 = -(1 + 2\psi)dt^2 + 2\partial_i \beta dt dx^i + a^2 (1 - 2\Phi) d\mathbf{x}^2. \tag{B1}$$

Expanding the scalar field as $\phi = \phi(t) + \delta\phi$, and considering a barotropic perfect fluid with equation of state $P = w\rho$ (for an action approach of perfect fluids see e.g. [23]), then we find that in Fourier space, the action at second order in the perturbation fields can be written as

$$\begin{aligned}
S^{(2)} = & \int dt d^3x a^3 \left\{ - \left(W_1 \psi + W_2 \dot{\delta\phi} - W_3 \dot{\Phi} - W_4 \delta\phi - \rho(1+w)V + W_5 \delta\chi \right) \frac{\partial^2 \beta}{a^2} + \frac{1}{2} \left(\frac{\rho(1+w)}{w} - W_6 \right) \psi^2 \right. \\
& - \left(W_7 \dot{\delta\phi} + W_8 \dot{\Phi} + \frac{\rho(1+w)(\dot{V} - 3wHV)}{w} - W_9 \frac{\partial^2 \delta\phi}{a^2} + W_{10} \frac{\partial^2 \Phi}{a^2} + W_{11} \delta\chi \right) \psi + \frac{1}{2} W_{12} \dot{\delta\phi}^2 + \frac{1}{2} W_{13} \dot{\Phi}^2 \\
& - \frac{1}{2} W_{14} \dot{\Phi} \dot{\delta\phi} - \frac{1}{2} W_{15} \frac{(\partial\delta\phi)^2}{a^2} - \frac{1}{2} W_{16} \frac{(\partial\Phi)^2}{a^2} + \frac{1}{2} \frac{\rho(1+w)}{w} \dot{V}^2 - \frac{1}{2} \frac{\rho(1+w)k^2 V^2}{a^2} - \frac{1}{2} f_{\chi\chi} \delta\chi^2 \\
& \left. + \left(W_{17} \dot{\delta\phi} - W_{18} \dot{\Phi} + W_{19} \frac{\partial^2 \Phi}{a^2} \right) \delta\chi - \left(-W_{20} \frac{\partial^2 \delta\phi}{a^2} - 9\rho w H(1+w)V + 3(1+w)\rho \dot{V} + W_{21} \dot{\delta\phi} \right) \Phi \right\}, \tag{B2}
\end{aligned}$$

where the matter field V is the scalar component of $\delta T^0_i = -\rho(1+w)\partial_i V$, so that the matter density contrast $\delta_m = \delta\rho/\rho$ can be written as $w\delta_m/(1+w) = \dot{V} - 3wHV - \psi$.

Notice we still have one gauge degree of freedom to choose. For example, we can consistently set $\beta = 0$ (Newtonian gauge), or $\delta\phi = 0$ (uniform field gauge), or $\Phi = 0$ (flat gauge).

The coefficients of the previous action are the following:

$$W_1 = 2 H M_{\text{pl}}^2 - 6 H \frac{c_G}{M^2} f_\chi \dot{\phi}^2, \quad (\text{B3})$$

$$W_2 = W_9 = 4 \frac{c_G}{M^2} f_\chi H \dot{\phi} \quad (\text{B4})$$

$$W_3 = W_{10} = -2 M_{\text{pl}}^2 + 2 \frac{c_G}{M^2} f_\chi \dot{\phi}^2, \quad (\text{B5})$$

$$W_4 = \dot{\phi} c_2 f_\chi + \dot{\phi} c_1 + 6 \dot{\phi} \frac{c_G}{M^2} H^2 f_\chi, \quad (\text{B6})$$

$$W_5 = 2 \frac{c_G}{M^2} f_{\chi\chi} H \dot{\phi}^2, \quad (\text{B7})$$

$$W_6 = -c_1 \dot{\phi}^2 + 6 M_{\text{pl}}^2 H^2 - 36 \frac{c_G}{M^2} f_\chi H^2 \dot{\phi}^2 - c_2 f_\chi \dot{\phi}^2, \quad (\text{B8})$$

$$W_7 = 18 \dot{\phi} \frac{c_G}{M^2} H^2 f_\chi + \dot{\phi} c_1 + \dot{\phi} c_2 f_\chi, \quad (\text{B9})$$

$$W_8 = 6 H M_{\text{pl}}^2 - 18 H \frac{c_G}{M^2} f_\chi \dot{\phi}^2, \quad (\text{B10})$$

$$W_{11} = f_{\chi\chi} \dot{\phi}^2 c_2 + 12 f_{\chi\chi} \phi_1^2 \frac{c_G}{M^2} H^2, \quad (\text{B11})$$

$$W_{12} = 6 \frac{c_G}{M^2} f_\chi H^2 + c_1 + c_2 f_\chi, \quad (\text{B12})$$

$$W_{13} = -6 M_{\text{pl}}^2 + 6 \frac{c_G}{M^2} f_\chi \dot{\phi}^2, \quad (\text{B13})$$

$$W_{14} = 24 \frac{c_G}{M^2} f_\chi H \dot{\phi}, \quad (\text{B14})$$

$$W_{15} = c_2 f_\chi + 4 \frac{c_G}{M^2} f_\chi \dot{H} + c_1 + 6 c_G f_\chi H^2, \quad (\text{B15})$$

$$W_{16} = -2 \frac{c_G}{M^2} f_\chi \dot{\phi}^2 - 2 M_{\text{pl}}^2, \quad (\text{B16})$$

$$W_{17} = \dot{\phi} f_{\chi\chi} c_2 + 6 \dot{\phi} f_{\chi\chi} \frac{c_G}{M^2} H^2, \quad (\text{B17})$$

$$W_{18} = 6 \frac{c_G}{M^2} f_{\chi\chi} H \dot{\phi}^2, \quad (\text{B18})$$

$$W_{19} = 2 f_{\chi\chi} \dot{\phi}^2 \frac{c_G}{M^2}, \quad (\text{B19})$$

$$W_{20} = \left[-4 \frac{c_G}{M^2} f_\chi - 4 \frac{c_G}{M^2} \dot{\phi}^2 (6 c_G H^2 + c_2) f_{\chi\chi} \right] \ddot{\phi} - 4 c_G f_\chi H \dot{\phi} - 24 \frac{c_G^2}{M^2} f_{\chi\chi} \dot{\phi}^3 H \dot{H}, \quad (\text{B20})$$

$$W_{21} = 3 \dot{\phi} \left(6 \frac{c_G}{M^2} f_\chi H^2 + c_1 + c_2 f_\chi \right). \quad (\text{B21})$$

The equations of motion for the perturbations in any gauge can be derived by using standard variational calculus.

2. Tensor perturbations

By introducing the two polarizations of transverse and traceless perturbations for the metric, we can write down the action expanded at second order as

$$S_{\text{GW}}^{(2)} = \sum_{\lambda=+, \times} \int dt d^3x a^3 \left[\frac{1}{8} \left(M_{\text{pl}}^2 - \frac{c_G}{M^2} f_\chi \dot{\phi}^2 \right) \dot{h}_\lambda^2 - \frac{1}{8} \left(M_{\text{pl}}^2 + \frac{c_G}{M^2} f_\chi \dot{\phi}^2 \right) \frac{(\partial h_\lambda)^2}{a^2} \right]. \quad (\text{B22})$$

This gives the no-ghost condition $1 - c_G f_\chi \bar{H}^2 x^2 > 0$, and speed of propagation equal to

$$c_{\text{GW}}^2 = \frac{1 + c_G f_\chi \bar{H}^2 x^2}{1 - c_G f_\chi \bar{H}^2 x^2}. \quad (\text{B23})$$

Note that due to the coupling to the Einstein tensor this is not equal to the speed of light. A stable evolution for the background requires that $c_{\text{GW}}^2 \geq 0$.

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